UNIT-2

(Lecture-7)

Design of Infinite Impulse Response Digital Filters: Chebyshev Filters

Raj Ranjan Prasad

Chebyshev Filters

The Chebyshev low-pass filter has a magnitude response given by

$$|H(j\Omega)| = \frac{A}{\left[1+\varepsilon^2 C_N^2(\Omega/\Omega_c)\right]^{0.5}} \quad \text{----------------(1)}$$

Where A is the filter gain, ε is a constant and Ω_c is the 3dB cut-off frequency. The Chebyshev polynomial of the I kind of Nth order, $C_N(x)$ is given by

$$C_N(x) = \begin{cases} \cos(N \cos^{-1} x), & \text{for } |x| \le 1 \\ \cosh(N \cosh^{-1} x), & \text{for } |x| \ge 1 \end{cases} \quad \text{------------(2)}$$

The design parameter of the Chebyshev filter are obtained by considering the low-pass filter with the desired specifications as given below

$$\delta_1 \le |H(e^{j\omega})| \le 1 \qquad 0 \le \omega \le \omega_1 \quad \text{---------(3a)}$$

$$|H(e^{j\omega})| \le \delta_2 \qquad \omega_2 \le \omega \le \pi \quad \text{----------(3b)}$$

Chebyshev Filters

The corresponding analog magnitude response is to be obtained in the design process. Using eq. (1) in eq. (3) and if A = 1,

$$\delta_1^2 \le \frac{1}{1+\varepsilon^2 C_N^2(\Omega_1/\Omega_c)} \le 1 \quad \text{--------------(4a)}$$

$$= \frac{1}{1+\varepsilon^2 C_N^2(\Omega_2/\Omega_c)} \le \delta_2^2 \quad \text{--------------(4b)}$$

Assuming $\Omega_c = \Omega_1$, we will have $C_N(\Omega_c/\Omega_c) = C_N(1) = 1$. Therefore, eq. (4a) can be written as $\delta_1^2 \le \frac{1}{1+\varepsilon^2}$

Assuming equality in the above equation, the expression for ε is

$$\varepsilon = \left[\frac{1}{\delta_1^2} - 1\right]^{0.5} \quad \text{-----------------(5)}$$

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Chebyshev Filters

The order of the analog filter, N can be determined from eq. (4b).

Assume $\Omega_c = \Omega_1$,

$$C_N(\Omega_2/\Omega_1) \geq \frac{1}{\varepsilon}\left[\frac{1}{\delta_2^2} - 1\right]^{0.5}$$

Since $\Omega_2 > \Omega_1$,

$$\cosh\left[N \cosh^{-1}(\Omega_2/\Omega_1)\right] \geq \frac{1}{\varepsilon}\left[\frac{1}{\delta_2^2} - 1\right]^{0.5}$$

or

$$N \geq \frac{\cosh^{-1}\left\{\frac{1}{\varepsilon}\left[\frac{1}{\delta_2^2} - 1\right]^{0.5}\right\}}{\cosh^{-1}(\Omega_2/\Omega_1)} \quad \text{------(6)}$$

The value of Ω_1 and Ω_2 are obtained using $\Omega = 2/T\,[\tan(\omega/2)]$ for bilinear transformation or $\Omega = \omega/T$ impulse invariant transformation.

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Chebyshev Filters

The coefficient b_k and c_k are given by

$$b_k = 2y_N \sin\left[(2k-1)\pi/2N\right]$$

$$c_k = y_N^2 + \cos^2 \frac{(2k-1)\pi}{2N}$$

$$c_0 = y_N \qquad \text{------------(7)}$$

The parameter y_N is given by

$$y_N = \frac{1}{2}\left\{\left[\left(\frac{1}{\varepsilon^2}+1\right)^{0.5} + \frac{1}{\varepsilon}\right]^{\frac{1}{N}} - \left[\left(\frac{1}{\varepsilon^2}+1\right)^{0.5} + \frac{1}{\varepsilon}\right]^{-\frac{1}{N}}\right\} \qquad \text{----(8)}$$

The parameter B_k can be obtained from

$$\frac{A}{(1+\varepsilon^2)^{0.5}} = \prod_{k=1}^{N/2} \frac{B_k}{c_k}, \text{ for } N \text{ even}$$

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Chebyshev Filters

and

$$A = \prod_{k=0}^{\frac{N-1}{2}} \frac{B_k}{c_k} \text{ for } N \text{ odd.}$$ ------------------(9)

The system function of the equivalent digital filter is obtained from H(s) using the specified transformation technique.